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AN ANALYTICAL METHOD FOR THE SYNTHESIS OF REVERSIBLE FOLLOW-UP SYSTEMS

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ABSTRACT. The technical and design requirements of reversible follow-up systems are presented. A synthesis scheme is proposed for the realization of these requirements.

Reversible follow-up systems for manipulators can, in the majority of cases, /83 be considered as a set of two interconnected systems, the first of which ensures copying the movements of the master arm by the slave arm, while the second reproduces the forces perceived by the slave arm at the master arm.

Designing reversible follow-up systems reduces to finding a reasonable compromise between attempts to obtain higher qualitative system indices and attempts to obtain these indices with simpler equipment. Extremely diverse synthesis methods are used to solve this problem. Synthesis can be treated as a problem of building a reversible follow-up system meeting a set of specifications, or as one of building an optimum system for specified limitations.

Given the state of the art, it is advisable to impose the following requirements on synthesis methods used to build reversible follow-up systems.

1. The synthesis methods should ensure the building of reversible follow-up systems with optimum dynamic characteristics for existing limitations on the power of slave drives and meeting specifications for all other indices. Moreover, planned systems should be simple in design, dependable in operation, and built of physically practicable elements.

2. Selection of main system elements should include their structural and energy characteristics. The energy characteristics of the main elements should provide specified limiting dynamic characteristics for the system as a whole.

3. Synthesis methods should derive from a specific, theoretically substantiated criterion of optimality, expressed perferably in unambiguous analytical form.

4. Synthesis methods should be as general-purpose as possible; that is, suitable for the design of single-circuit and multicircuit systems, autonomous and interconnected systems, linear systems and systems with a limited zone of

linearity, and the like.

5. The synthesis method should provide an opportunity for the optimization of follow-up system regulation laws, and an opportunity to optimize control.

As a rule, reversible follow-up systems should meet the following specifications. /84

1. The follow-up systems should provide smooth movements of the manipulator slave arm for specified speeds and accelerations.

2. The systems should accurately reproduce at the master arm force perceived by the slave arm. For ease of control, the friction forces generated in the kinematic chain of manipulator master and slave arms should be reduced to the minimum, or should be adequately compensated for by using the corresponding reactions on the slave drives.

3. The systems should ensure specified follow-up precision.

4. The systems should have optimum dynamic characteristics under conditions of limiting the power of the slave drives and power sources.

5. The systems should have design characteristics corresponding to specified manipulator operating conditions.

The following synthesis scheme is proposed for the realization of the above requirements.

The type and parameters of the slave drives are determined in the first stage of the design of a reversible follow-up system. The properties of the drives usually limit the opportunities for improving the quality of adjustment by specific limits. The design and power characteristics of the drives should correspond to specifications. The dynamic characteristics of follow-up systems with the above-indicated drives should be optimum. The limiting dynamic characteristics of the follow-up system must be evaluated when selecting slave drives to compare different possible variants. The selection of the slave drives predetermines the remaining functionally necessary elements in the system and permits compiling a preliminary block diagram and determining the order of the differential equation which characterizes the motion of the system under transient and steady-state operating conditions.

Optimization criteria are selected, and the parameters of the desired transfer function, or desired equation of motion, providing the required indices of the optimization criteria are developed in the second stage of design.

Correction devices providing the specified parameters for the optimum system are developed in the third stage.

The scope of this article does not permit examining all stages of the designing of a reversible follow-up system, so we shall limit ourselves to some of the problems of the synthesis of follow-up systems operating under definite, concrete, conditions.

Let us assume it to be necessary to create a system which is optimum in speed of operation in the limit mode for reproduction of a suddenly applied, abrupt input. The possibilities of increasing operating speed in real systems are limited by the magnitude of the maximum moment developed by the slave motor during the reproduction of a given trajectory of motion. The optimum system operating speed is reached when its slave motor, under the most difficult follow-up mode develops the maximum rotary moment in the required direction for the whole of the transient. Observance of this condition ensures maximum operating speed and minimum power consumption to reproduce a given trajectory. The curve computation is not particularly difficult if the input controlling influence is given. The specified transient curve for the most characteristic controlling influence and which considers all limitations introduced by the power section of the follow-up system can be taken as the basis for the synthesis of the optimum follow-up system. The criterion of optimality in this case is the degree of approximation of the transient curve reproduced by the system to the optimum curve calculated for the same controlling influence. /85

The optimum transient curve for an abrupt input can be calculated using the formulas in Table 1.

TABLE 1

t	$\Omega_{opt}(t)$	$S_{\varphi opt}(t)$
$0 < t < \xi_2 t_r$	$a_1 t$	$\frac{1}{2} a_1 t^2$
$\xi_2 t_r < t < t_r$	$a_2 (t_r - t)$	$a_2 t_r t - \frac{1}{2} a_2 t^2 - - \frac{1}{2} a_2 \xi_2 t^2 r$

where

a_1 is the maximum acceleration during motor speedup;

a_2 is the maximum acceleration when the motor is braked;

t_r is the regulating time (transient time);

Ω is the angular velocity;

$\xi_1 = a_1/(a_1 + a_2)$;
$\xi_2 = a_2/(a_1 + a_2)$.

The optimum trajectory also can be presented in the form of a trigonometric sum:

$$S_{\varphi_{opt}}(t) = A_{0_{opt}} + \sum_{k=1}^{\nu} (A_{k_{opt}} \cos k\omega t + B_{k_{opt}} \sin k\omega t), \quad (1)$$

where k = 1, 3, 5, 7...

$$A_{k_{opt}} = \frac{16}{(\pi k)^2}$$
$$B_{k_{opt}} = \frac{16}{(\pi k)^3} \varphi_{ss}, \quad (2)$$

$A_{o_{opt}} = \frac{1}{2} S_{\varphi_{ss}}$; $S_{\varphi_{ss}}$ is the steady-state value of $S_\varphi(t)$.

From Table 1

(3)

Maximum accelerations a_1 and a_2 are calculated from the formulas

(4)

where

M_{max} is the motor's maximum torque;

M_r is the normalized moment of resistance;

J'_Σ is the summed normalized moment of inertia of the drive.

The M(t), a(t), Ω(t), and $S_\varphi(t)$ curves for the optimum transient state are

shown in Figure 1.

Eqs. (3) and (4) characterize the interconnection between the power characteristics of the slave drive and the maximum dynamic curves for an optimum follow-up system. The preliminary selection of the motor is made from Eqs. (3) and (4) obtained from the condition of attainment of optimum operating speed. After /86
selection of the motor, a block diagram of the system is compiled from the functionally necessary elements and the order of the differential equation of its motion is determined.

The main task of the follow-up system synthesis is formulated as follows. Necessary for a stable system whose motion is described by the equation

$$D(p)\,S_{\varphi}(t) = R(p)\,S_{\varphi_g}(t) - N(p)\,M(t), \tag{5}$$

where

$D(p) = p^n + b_1 p^{n-1} + \ldots + b_{n-1} + b_n$;

$R(p) = p^m + c_1 p^{m-1} \ldots + c_m$;

$N(p) = P^v + d_1 p^{v-1} + \ldots d_v$;

$S_{\varphi_g}(t)$ is the trajectory given for reproduction;

$M(t)$ is the generalized disturbing force among all allowable parameters $b_1, b_2, \ldots, b_n$, is determination of those for which the trajectory $S_{\varphi}(t)$ would differ from some optimum trajectory $S_{\varphi opt}(t)$ by the amount of allowable error.

An analytical solution of this problem is possible by using the approximate direct method of solving differential equations proposed by the author and called the periodic solutions method. The method is based on the following assumption: The trajectory of a stable self-regulating system, $S_{\varphi}(t)$, fixed in a finite time segment $0 < t < t_r$, during the transient for a given action $S_{\varphi_g}(t)$, can be reproduced by this same system in the same time segment for the corresponding steady-state periodic motion $S_{\varphi_p}(t)$ if the following conditions are observed: (1) the controlling influences for the transient $S_{\varphi_g}(t)$ and for the steady-state periodic motion $S_{\varphi_{gp}}(t)$ coincide on time segment $0 < t < t_r$; (2) the period of

periodic influence $T \gg 4t_r$; (3) outside time segment $0 < t < t_r$ the periodic influence $S_{\varphi_{gp}}(t)$ should change in such a way as to ensure the practical identity of the initial conditions for t = 0 for transient and periodic motions (Figure 2).

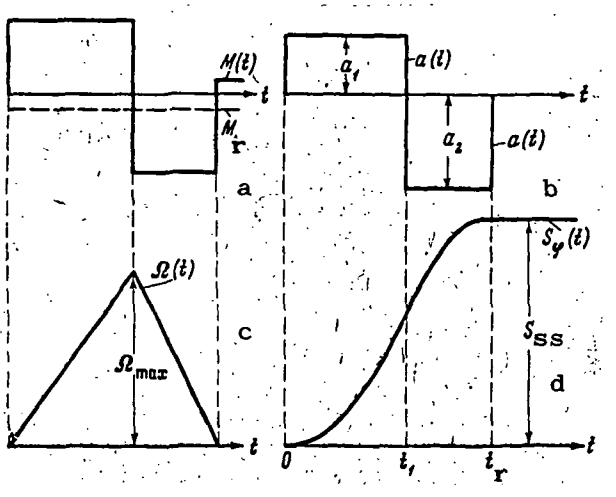

Figure 1. Curves of an optimum transient state a - M(t); b - a(t); c - $\Omega(t)$; d - $S_\varphi(t)$.

The practical coincidence of motion trajectories $S_\varphi(t)$ and $S_{\varphi p}(t)$ on segment $0 < t < t_r$ permits replacing the corresponding cumbersome and complicated method of analyzing the transients of self-regulating systems by the periodic solutions method.

As an example, let us determine the motion trajectory for a system described by Eq. (5) by the periodic solutions method, considering that the given equation is linear. Let us replace the nonperiodic actions in the right-hand side of the Eq. $R(p)S_{\varphi_g}(t) - N(p)M(t)$ with the corresponding periodic action, given in the form of the trigonometric series

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(6)

where

$\omega = 2\pi/T$;

T is the period of the action;

C_0, C_k, D_k are constants determined from Euler-Fourier formulas.

We will seek the solution of Eq. (5) with a changed right member as the sum of the functions

Each of these functions, taken separately, should satisfy Eq. (5). Substituting the functions $S_{\varphi_{ka}}(t)$, $S_{\varphi_{kb}}(t)$, $S_{\varphi_0}(t)$ into Eq. (5) with the right-hand side of

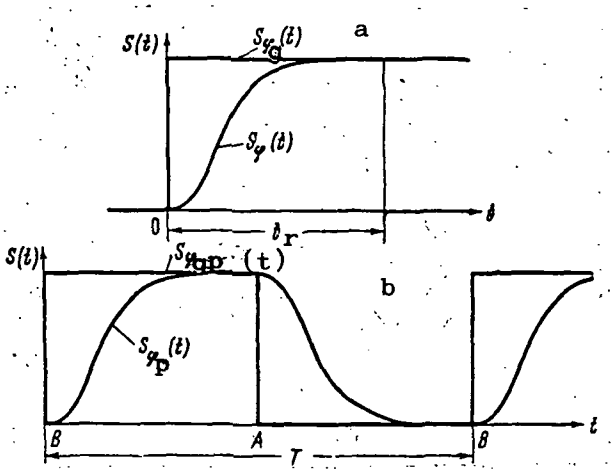


Figure 2. Trajectories of motion in the transient (a) and periodic (b) states.

Eq. (6), and comparing the coefficients when the time functions are the same, we determine the values of A_k and B_k

(7)

where

$$\lambda_{1k} = \frac{F_{1k}}{F_{1k}^2 + F_{2k}^2} \tag{8}$$

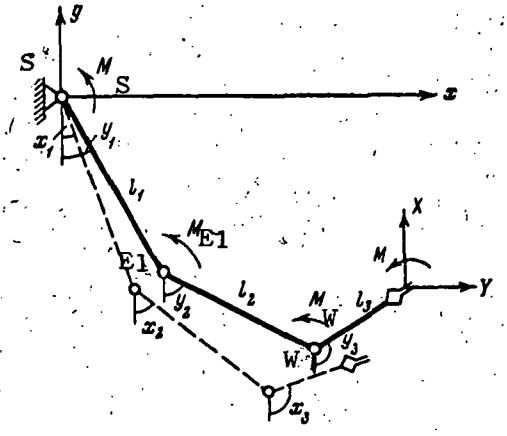
$$\lambda_{2k} = \frac{F_{2k}}{F_{1k}^2 + F_{2k}^2}. \tag{9}$$

The functions F_{1k} and F_{2k} can be determined by using the formulas in Table 2.

The trajectory reproduced by a system for the corresponding periodic motion is determined by the trigonometric sum

$$S_{\varphi p}(t) = A_0 + \sum_1^{\nu} A_k \cos k\omega t + \sum_1^{\nu} B_k \sin k\omega t; \quad S_{\varphi}(t) \approx S_{\varphi_p}(t). \tag{10}$$

Now let us move on to the solution of the main problem of synthesis. Let us assume that the optimum trajectory of system motion, Eq. (5), over a segment $0 < t < t_r$ is defined by Eq. (1) and that the actual system motion trajectory is determined by Eq. (10). Appearing with the equality $S_{\varphi_p}(t) \approx$ /88
$\approx S_{\varphi_{opt}}(t)$ is the following analytical connection between the parameters of Eq. (5) $b_1, b_2, \ldots, b_n$ and the parameters of the optimum transient curve, described by Eq. (5)



(11)

where

$$\varphi_{1k} = \left[D_k \frac{A_{k\,opt}}{A^2_{k\,opt} + B^2_{k\,opt}} - C_k \frac{B_{k\,opt}}{A^2_{k\,opt} + B^2_{k\,opt}} \right];$$
$$\varphi_{2k} = \left[D_k \frac{B_{k\,opt}}{A^2_{k\,opt} + B^2_{k\,opt}} + C_k \frac{A_{k\,opt}}{A^2_{k\,opt} + B^2_{k\,opt}} \right]. \qquad (12)$$

TABLE 2

order of equation n	F_{1k}	F_{2k}
2	$-[(k\omega)\, b_1]$	$-[(k\omega)^2 - b_2]$
3	$[(k\omega)^3 - (k\omega)\, b_2]$	$-[(k\omega)^2 b_1 - b_3]$
4	$[(k\omega)^3 b_1 - (k\omega)\, b_3]$	$[(k\omega)^4 - (k\omega)^2 b_2 + b_4]$
5	$-[(k\omega)^5 - (k\omega)^3 b_2 + (k\omega)\, b_4]$	$[(k\omega)^4 b_1 - (k\omega)^2 b_3 + b_5]$
6	$-[(k\omega)^5 b_1 - (k\omega)^3 b_3 + (k\omega)\, b_5]$	$-[(k\omega)^6 - (k\omega)^4 b_2 + (k\omega)^2 b_4 - b_6]$
7	$[(k\omega)^7 - (k\omega)^5 b_2 + (k\omega)^3 b_4 - (k\omega)\, b_6]$	$-[(k\omega)^6 b_1 - (k\omega)^4 b_5 + (k\omega)^2 b_5 - b_7]$
8	$[(k\omega)^7 b_1 - (k\omega)^5 b_3 + (k\omega)^3 b_5 - k\omega\, b_7]$	$[(k\omega)^8 - (k\omega)^6 b_2 + (k\omega)^4 b_4 - (k\omega)^2 b_6 + b_8]$

This analytical connection is presented in developed form for differential equations of various orders by the formulas in Table 3. Systems with parameters b_1, b_2 ..., b_n, calculated using the formulas in Table 3, will reproduce the given optimum motion trajectory accurately to the ν^{th} harmonic. $\nu = n/2$ for even values of n; $\nu = (n-1)/2$ for odd values of n. The derived differential equation for the system is checked for stability after the calculated determination of the coefficients of the characteristic equation b_1, b_2, ..., b_n. If the system is unstable, it cannot reproduce the designated optimum motion trajectory so this trajectory must be changed. In the case of an abrupt controlling action for example, the optimum curve, determined using the formulas in Table 1, or the Eq. 1 trigonometric sum, is reproduced by the follow-up system whose order of differential equation of motion is $n \leq 4$. For systems with an equation of motion of a higher order, an attempt to reproduce the above-indicated optimum curve upset motion stability; that is, the maximum speed of operation of the system in this case is limited by the potentials of the slave drive, as well as by the properties of other functionally necessary elements. The problem of synthesizing a system with optimum operating speed when an adequate margin of stability is available is also solved when the system's characteristic equation is presented in the form of the product of several equations whose order is $n \leq 4$. Each of these equations can be synthesized

separately by the method presented above. As an example, let us consider the characteristic seventh order equation, which can be presented in the form of the product of two third and fourth order equations

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$$D_7(p) = p^7 + b_1p^6 + \ldots + b_6p + b_7 = D_3(p)\,D_4(p) = (p^3 + b_{13}p^2 + \ldots + b_{33})(p^4 + b_{14}p^3 + \ldots + b_{34}p + b_{44}).$$

TABLE 3

n	Coefficient of optimum system equation	n	Coefficient of optimum system equation
2	$b_1 = -\frac{\varphi_{11}}{1-\varphi_{21}}\omega = \frac{\varphi_{11}}{Q_2}\omega$ $b_2 = \frac{1}{Q_2}\omega^2$	4	$b_1 = 3\frac{\varphi_{13}-3\varphi_{11}}{Q_4}\omega;\ Q_4 = 8+\varphi_{23}-9\varphi_{21}$ $b_2 = \frac{80+\varphi_{23}-81\varphi_{21}}{Q_4}\omega^2$
3	$b_1 = 24\frac{1-\varphi_{21}}{Q_3}\omega;\ Q_3 = \varphi_{13}-3\varphi_{11}$ $b_2 = \frac{\varphi_{13}-27\varphi_{11}}{Q_3}\omega^2$		$b_3 = 3\frac{\varphi_{13}-27\varphi_{11}}{Q_4}\omega^2$ $b_4 = \frac{72}{Q_4}\omega^4$
	$b_3 = 24\frac{1}{Q_3}\omega^3$	6	$b_1 = \frac{-2\varphi_{11}+\varphi_{13}-\frac{1}{5}\varphi_{15}}{Q_6}\omega$
5	$b_1 = 16\frac{3(1-\varphi_{21})-\frac{1}{3}(1-\varphi_{23})}{Q_5}\omega$ $b_2 = \frac{26\varphi_{13}-68\varphi_{11}-2\varphi_{15}}{Q_5}\omega^2$ $b_3 = \frac{81(1-\varphi_{21})-(1-\varphi_{23})}{Q_5}16\omega^3$ $b_4 = \frac{25\varphi_{13}-450\varphi_{11}-\frac{9}{5}\varphi_{15}}{Q_5}\omega^4$ $b_5 = \frac{384}{Q_5}\omega_5$ $b_5 = \varphi_{13}-2\varphi_{11}-\frac{1}{5}\varphi_{15}$		$b_2 = \frac{68(1-\varphi_{21})-\frac{26}{3}(1-\varphi_{23})+\frac{2}{15}(1-\varphi_{25})}{Q_6}\omega^2$ $b_3 = \frac{-68\varphi_{11}+26\varphi_{13}-2\varphi_{15}}{Q_6}\omega^3$ $b_4 = \frac{450(1-\varphi_{21})-\frac{25}{3}(1-\varphi_{23})+\frac{9}{25}(1-\varphi_{25})}{Q_6}\omega^4$ $b_5 = \frac{-450\varphi_{11}+25\varphi_{13}-\frac{9}{5}\varphi_{15}}{Q_6}\omega^5$ $b_6 = \frac{384}{Q_6}\varphi^6$ $Q_6 = 2(1-\varphi_{21})-\frac{1}{3}(1-\varphi_{23})+\frac{1}{25}(1-\varphi_{25})$

We will consider the fourth order equation to be the basic one. The

parameters of this equation can be determined using the formulas in Table 3 to include the specified regulation time t_r. A third order equation takes into account the additional influence on the transient of the links of a system with smaller time constants. The parameters of this equation too are determined using the formulas in Table 3, but include regulation time $t_{r1} < t_r$. The smaller t_{r1} as compared to t_r, the closer the system transient curve will be to the specified optimum curve. Stability of motion will be preserved for any values of t_{r1} and t_r. Knowing the coefficients of the third and fourth order /90
equations, it is easy to determine the coefficients of a seventh order equation. The proposed method of synthesis is applicable in all cases when the input actions causing the system transient are predetermined. The synthesis therefore should be preceded by investigations to determine the most characteristic standard, or maximum, operating conditions for the follow-up system for the existing working conditions.

The proposed synthesis method also is applicable to some degree to nonlinear follow-up systems if all the nonlinear links can be combined into a common nonlinear link, structurally linked to the linear part of the system.

Let us examine Eq. (13)

(13)

where

F(S, pS, t) is a nonlinear function of the variable $S_\varphi(t)$ and of its derivative.

We will say that the function $S_{\varphi_g}(t)$, M(t), $S_\varphi(t) \approx S_{\varphi_{opt}}(t)$ is given. In this case the function F(S, p, S, t) can be determined without particular difficulty.

Let us present the variables $S_{\varphi_g}(t)$, M(t), $S_{\varphi_{opt}}(t)$, and F(S, p, S, t) in the form of a finite sum of sinusoidal functions

$$S_{\varphi_{opt}}(t) \approx \left[A_{0_{opt}} + \sum_1^{\nu} (A_{k_{opt}} \cos k\omega t + B_{k_{opt}} \sin k\omega t)\right];$$

$$R(p) S\varphi_g(t) - N(p) M(t) \approx \left[C_0 + \sum_1^{\nu} (C_k \cos k\omega t + D_k \sin k\omega t)\right] b_n S_{\varphi_{ss}}; \quad (14)$$

$$F(S, pS, t) = F_1(t) \approx \left[f_0 + \sum_1^{\nu} (f_k \cos k\omega t + h_k \sin k\omega t)\right] b_n S\varphi_{ss}.$$

Substituting each of the sinusoidal functions into Eq. (13) with the changed right-hand side and comparing the coefficients when the time functions are the same, we determine the auxiliary functions φ_{1k} and φ_{2k}

$$\varphi_{1k} = D_{k\Sigma}\frac{A_{k\,opt}}{A^2_{k\,opt}+B^2_{k\,opt}} - C_{k\Sigma}\frac{B_{k\,opt}}{A^2_{k\,opt}+B^2_{k\,opt}};$$
$$\varphi_{2k} = D_{k\Sigma}\frac{B_{k\,opt}}{A^2_{k\,opt}+B^2_{k\,opt}} + C_{k\Sigma}\frac{A_{k\,opt}}{A^2_{k\,opt}+B^2_{k\,opt}}, \qquad (15)$$

where

$$D_{k_\Sigma} = D_k + h_k;$$

$$C_{k_\Sigma} = C_k + f_k.$$

The parameters of the linear part of the system, b_1, b_2, ..., b_k, are determined using the formulas in Table 3.

The point of the proposed nonlinear equation synthesis method is that in essence a linear equation is synthesized, but the influence of the nonlinearity of the curves for some links can be included by the introduction of the corresponding additional disturbing actions.

Another formulation of the basic synthesis task is possible.

What must be found for a stable automatic regulation system such as Eq. (5) for specified parameter values and specified disturbing actions is that control, $S_{\varphi_g}(t)$, for which the motion trajectory, $S_\varphi(t)$, would differ from the given /91 optimum motion trajectory, $S_{\varphi_{opt}}(t)$, by no more than the value of the permissible error.

Using the method of periodic solutions for this case, we can obtain the following analytical relationships

$$C_{0_{opt}} = \frac{1}{S\varphi_{ss}}(A_{0_{opt}} + m_0 M);$$
$$C_{k_{opt}} = \frac{1}{b_n S\varphi_{ss}}[(A_{k_{opt}}F_{2k} - B_{k_{opt}}F_{1k}) + m_k M]; \qquad (16)$$
$$D_{k_{opt}} = \frac{1}{b_n S\varphi_{ss}}[(A_{k_{opt}}F_{1k} + B_{k_{opt}}F_{2k}) + n_k M].$$

The functions F_{1k} and F_{2k} are deciphered in Table 2. The non-periodic disturbing action in Eq. (5) is replaced by a periodic disturbing action and is expanded in a trigonmetric function series

$$N(p)M(t) = \left[m_0 + \sum_1^{\nu}(m_k \cos k\omega t + n_k \sin k\omega t)\right] M \cdot b_n. \qquad (17)$$

Finally

$$S\varphi_{opt}(t) = \left[C_{0\,opt} + \sum_1^{\nu}(C_{k\,opt}\cos k\omega t + D_{k\,opt}\sin k\omega t)\right] S_{\varphi g}. \qquad (18)$$

Along with optimum dynamic characteristics, reversible follow-up systems should have the required precision of follow-up and the required precision of force reflection. There are different ways to increase the regulation precision. In some cases, the system gain is increased, but its dynamic characteristics change significantly as a result. The most effective method of increasing regulation precision without significant change in dynamic characteristics is the method that uses an additional isodromic element to increase regulation precision and to compensate for unneeded friction forces.

The concluding stage of the automatic regulation system synthesis is the selection of the correctors and the determination of their parameters. The correctors should ensure optimum values of system parameters [(the coefficients b_1, b_2, ..., b_n in Eq. (5)] or optimum values of control paramters [(the coefficients $C_{k_{opt}}$, $D_{k_{opt}}$ in Eq. (18)]. Preference in the selection of correctors should be given to direct regulating feedbacks because they do not raise the order of the differential equation of system motion, reduce the influence of nonlinearities, ensure stability of characteristics, or increase operating speed, nor do they require complex additional elements for practical realization.

In the motion of a follow-up system can be characterized by a n^{th} order differential equation, then the number of independently acting correctors necessary for complete optimization of characteristics is equal to n. The number of independent correctors necessary to obtain a specified form of transient curve can be one or two less.

Some correctors are already installed in the initial block diagram of the

system made up of functionally necessary elements. We can, in systems with an isodromic element ensuring given regulation precision under steady-state operating conditions, use a common system amplification factor and isodromic element time constant as the correctors. Both values submit to change readily. Correctors /92 that raise the order of the system differential equation are unsuitable in this case because they do not close the gap between the number of correctors necessary to attain the optimum transient and the number of correctors actually installed in the system. The parameters of the correctors selected are calculated as follows. The coefficients of differential Eq. (5), b_{10}, b_{20}, ..., b_{n0}, corresponding to the optimum transient are determined using the formulas in Table 3. The equations for coefficients b_1, b_2, ..., b_n are determined as a function of the variable and invariable system parameters and links from the block diagram of the system including the selected correctors. Corrector parameters are calculated using equations which derived as a result of comparing coefficients b_{10}, b_{20}, ..., b_{n0} with coefficients b_1, b_2, ..., b_n. These equations, as a rule, are solvable relative to the magnitudes of interest to us.

Let us dwell on the specifics of the synthesis of interconnected systems.

Let us consider the reversible follow-up system with independent control channels, the block diagram of which is shown in Figure 3a. In this case, we have a set of two regulating systems.

The first regulating system, consisting of amplifier 1, the drive (blocks 2, 3, 4, 5, 6), and position sensors 7, 15, is the follow-up system.

The second regulating system, consisting of amplifier 9, the drive (blocks 10, 11, 12, 13, 14), and force sensor 8 is intended for transmitting forces from the slave arm to the master arm.

Let us consider the equations of motion of this reversible system. The master arm drive is

$$pS_{m_m}\left(p+\frac{1}{T_m}\right)=\frac{k_{am}}{c_{emf_m}T_m}u_{\delta m}+k_{F_m}F_m-k_{m_m}M'_{\dot{u}_m}. \qquad (19)$$

The slave arm drive is

(20)

$$[(J_2+J_0)p+(c_2+c_0)]\,p\bar{y}=\bar{M}_2+\bar{M}_0.$$

$$\left.\begin{aligned} X\xi_1 + Y\eta_1 &= M_S - M_{E2}; \\ X\xi_2 + Y\eta_2 &= M_{E1} - M_W; \\ X\xi_3 + Y\eta_3 &= M_W - M_T; \end{aligned}\right\}$$

where

S_{mm} and S_{ms} are the angles of rotation of the output shafts of the motors of the master and slave arms;

T_m and T_s are the electromechanical time constants of the drives;

K_{am} and K_{as} are the amplification factors of the respective amplifiers;

$C_{emf\ m}$ and $C_{emf\ s}$ are the coefficients of the emf of the motors;

K_{Fm} and K_{Fs} are the coefficients of the forces;

K_{mm} and K_{ms} are the coefficients of the moments;

M'_{ism} and M'_{iss} are the summed idling moments attributable to friction forces.

The position sensors are

$$u_{\delta s} = k_{\delta S}(S_{mm} - S_{ms}). \tag{21}$$

The force sensors are

$$u_{\delta m} = k_{\delta F}(F_m - F_s), \tag{22}$$

where

F_s and F_m are the forces acting on the slave and master arm;

$K_{\delta S}$ is the position coefficient;

$K_{\delta F}$ is the force coefficient.

Solving Eqs. (19) - (22) simultaneously, we obtain Eqs. (23) and (24)

$$\left(p^2 + \frac{1}{T_s}p + \frac{k_{as}k_{\delta S}}{c_{emfs}T_s}\right)S_{ms} = \frac{c_{emfs}T_m k_{Fs}}{k_{am}k_{\delta F}}\left[p^2 + \frac{1}{T_m}p + \frac{k_{am}k_{\delta F}k_{as}k_{\delta s}}{c_{emfs}T_m k_{Fs}c_{emfs}T_{emfs}}\right]S_{mm} - \\ - k_{Fs}(1 + \psi_{Fm})F_m - k_{ms}M'_{iss} + k_{mm}M'_{ism}; \tag{23}$$

$$pS_{ms}\left[p^3 + \left(\frac{1}{T_m} + \frac{1}{T_s}\right)p^2 + \left(\frac{1}{T_m T_s} + \frac{k_{as}k_{\delta S}}{c_{emfs}T_s}\right)p + \frac{k_{as}k_{\delta s}}{c_{emfs}T_s T_m}\right] = \\ = \frac{k_{as}k_{am}k_{\delta S}k_{\delta F}}{c_{emfs}c_{emfm}T_s T_m}[(1 + \psi_{Fm})F_m - F_s] - \frac{k_{as}k_{\delta S}}{c_{emfs}T_s}k_{mm}M'_{ism} \tag{24}$$

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where

$$\psi_{Fs} = \frac{k_{Fs}c_{emf\ m}T_s}{k_{am}k_{\delta_F}}$$

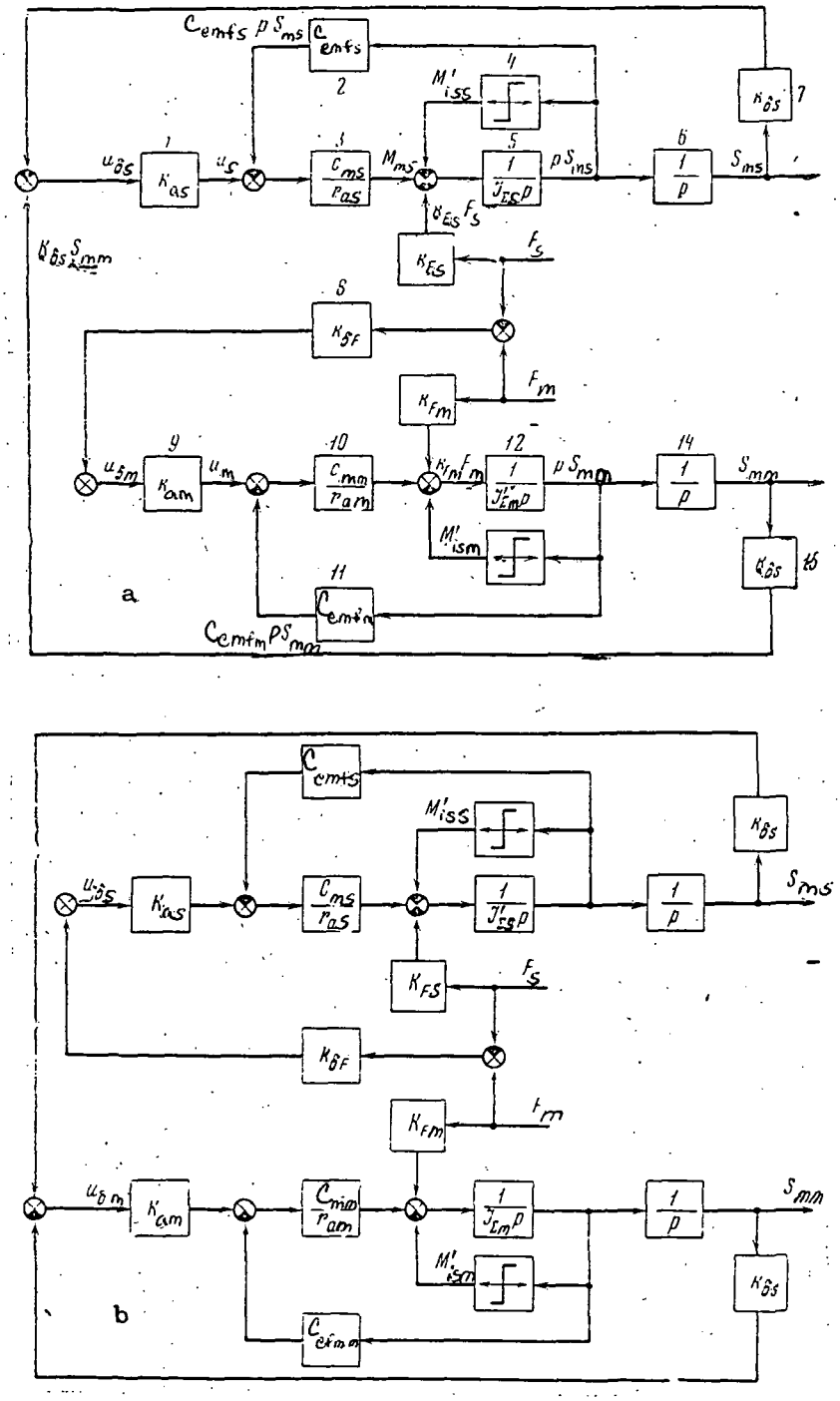

Figure 3. Reversible follow-up systems with independent control channels. a - first variant; b - second variant.

Another coupling arrangement also is possible (Figure 3b). In this case

(25)

Solving Eqs. (19), (20), and (25) simultaneously, we obtain

$$S_{mm}\left(p^2+\frac{1}{T_m}p+\frac{k_{am}k_{\delta S}}{c_{emfm}T_m}\right)=\frac{k_{am}k_{\delta S}}{c_{emfm}T_m}S_{ms}+k_{Fm}F_m-k_{mm}M'_{iSm} \tag{26}$$

$$pS_{ms}\left(p+\frac{1}{T_S}\right)=\frac{k_{as}k_{\delta F}}{c_{emfs}T_S}[F_m-(1+\psi_{FS})]-k_{ms}M'_{iss}, \tag{27}$$

where

$$\psi_{FS}=\frac{k_{FS}c_{emfs}T_S}{K_{as}k_{\delta F}}.$$

A detailed consideration of Eqs. (23), (24), and (26), (27) leads to the following conclusions.

1. The drives for follow-up and force reflection are interconnected in the reversible follow-up system (see Figure 3a), but in the reversible follow-up system (see Figure 3b) they are autonomous.

2. A variable load, and a variable slave arm moment of inertia, have a significant influence on the operation of the follow-up drive in a reversible follow-up system (see Figure 3a), and follow-up precision and the dynamic characteristics of the follow-up drive in the system (see Figure 3b) are determined uniquely by the parameters of the master arm.

Thus, the second variant of the block diagram (see Figure 3b) is the more advantageous.

Eqs. (23), (24) and (26), (27) can be synthesized in accordance with the method presented above to obtain a reversible system optimum in speed of operation and built according to the diagram (see Figure 3).

If an isodromic element is added to the force reflection system (see Figure 3b) the influence of the idling moments is additionally compensated for and Eq. (27) will have the following form

$$p^2S_{ms}\left(p+\frac{1}{T_S}\right)=\frac{k_{as}k_{\delta F}}{c_{emfs}T_S}[(F_m-F_S)_p+k_{ie}(F_m-F_S)] \tag{28}$$

when $M'_{is\ s}$ = const, where K_{ie} is the coefficient of the isodromic element. In the steady-state operating mode

$$F_m = F_s. \tag{29}$$

The advantages noted here of a system built in accordance with the block diagram (see Figure 3b), as compared to the system in Figure 3a, also are retained in more complex reversible systems with independent control channels. Conditions for the independence of regulating systems are ensured when the position sensors are connected directly to the shafts of the slave motors and when the dynamic forces acting on the force sensor of the master arm can be ignored in practice.

Conclusions /95

The investigations have established that:

1. The maximum operating speed of follow-up systems is limited by the properties of the power elements, which usually are electric or hydraulic drives.

2. Follow-up system drives optimum in operating speed should develop maximum accelerations in the required direction throughout the transient in limit modes.

3. Optimum dynamic characteristics of systems operating in limit modes are determined from sufficiently simple calculation equations and known power curves of preselected slave drives.

Thus, criteria exist which permit the comprehensive evaluation of each selected drive even prior to designing the follow-up system as a whole.

4. The use of the proposed periodic solutions method can establish a simple, unambiguous analytical connection between the optimum transient curve fixed in the limit mode with a given input action, and with the parameters of the follow-up system reproducing the optimum curve. This relationship permits determining the parameters of an optimum system, or its optimum control.

The proposed method of follow-up systems synthesis based on the above-indicated analytical relationships is sufficiently universal; that is, equally suitable for designing linear and some types of nonlinear systems, single-circuit and multi-circuit systems, etc.

The method provides for the building of optimum systems when there are limitations on the capacity of power elements and power supply sources.

The use of isodromic elements, and of other known equipment, makes it

possible to increase significantly follow-up precision and accuracy in the reflection of forces.